Autoregressive conditional parameter model with heteroskedastic regressors

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Abstract

To do with the ARCH effects in explanatory variables, a new time-varying parameter regression is developed. The autoregressive conditional parameter (ACP) model with heteroskedastic regressors extends the ACP model of Lu and Wang (2016) by allowing explanatory variables to follow a multivariate GARCH process. The model is applied to examine time-varying causal effects of the daily United States (US) dollar exchange rate and S&P 500 stock index on WTI crude oil price. The empirical results show that the developed model outperforms the linear regression and ACP model. The casual effects of US dollar and S&P 500 stock indices on WTI are time-varying and become stronger after 2008.

Keywords: autoregressive conditional parameter model, heteroskedastic regressor, ARCH effect, crude oil

JEL: C22, C50

1. Introduction

Time-varying parameter models are useful to detect the evolving economic and financial systems. Many methods have been proposed and widely applied in empirical studies. One popular method may be the so-called time-varying parameter model, where the time-varying parameter follows a random walk process (e.g., Kalman, 1960; Sims, 1989; Cogley and Sargent, 2005; Primiceri, 2005). Another is the regime-switching model, where the time-varying parameter follows a nonlinear regime-switching process (e.g., Tsay, 1989; Hamilton, 1989; Cai, 1994; Hamilton and Susmel, 1994).

Recently, Lu and Wang (2016) proposed a new, simple time-varying conditional parameter model, i.e., the autoregressive conditional parameter (ACP) model. It's convenient to identify, test, and estimate the ACP model. For example, the orders of ACP model can be easily identified using the autocorrelation function (ACF) and partial autocorrelation function

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Any comments and suggestions for improvement are welcome and gratefully appreciated. The corresponding author takes full responsibility for any errors and shortcomings in the paper.

(PACF). The empirical results showed that the ACP model outperforms the traditional linear regression, and the presence of ACP effect was found. But the explanatory variables in ACP model are assumed to be identical independent distributed (i.i.d.), which may not hold in the empirical studies.

Meanwhile, heteroskedasticity or ARCH effect has been widely found in economic and financial time series (for instances, Engle, 1982; Bollerslev, 1986; Nelson, 1991; Bollerslev et al., 1992; Bollerslev and Engle, 1993; Ederington and Guan, 2013; Engle and Sheppard, 2001; Engle, 2002). Engle (1982) found that quarterly British consumer price index presented a significant ARCH effect. Using daily returns for the value-weighted market index from the CRSP tapes, Nelson (1991) confirmed the existence of ARCH effect. Engle (2002) proposed the dynamic conditional correlation (DCC) GARCH model, and the empirical results showed some correlations among some financial markets are time-varying. Ederington and Guan (2013) applied the EGARCH model of Nelson (1991) for 43 daily financial returns and confirmed the presence of ARCH effect.

The presence of ARCH effect may impact the efficiency of the estimate, the power of a test, and the performance of a model (e.g., White, 1980; Dufour and Taamouti, 2010). Particularly, the ACP model of Lu and Wang (2016) is not applicable when heteroskedastic explanatory variables are used, because the i.i.d assumption of explanatory variables is violated. Therefore, this paper extends the ACP model to allow for heteroskedastic regressors by assuming the explanatory variables to be a multivariate GARCH process.

The model is applied to study time-varying effects of daily US dollar index and S&P 500 stock market on WTI crude oil price. Lots of empirical studies have examined their relations since Hamilton (1983) showed crude oil price shock was a factor in the US recession. But the empirical findings were complex and sometimes contradictory. For example, in the relations between crude oil price and US dollar exchange rate, some researchers found that the exchange rate impacted oil prices (e.g., Pindyck and Rotemberg, 1991; Sadorsky, 2000; Akram, 2009). But some showed no significant effect of US dollar on crude oil price (e.g., Norden, 1998; Zhang et al., 2008). Time-varying effect of US dollar on crude oil has also been found (e.g., Harris, 1995; Wu, Chung, and Chang, 2012). In the causal relations between crude oil and stock markets, Ciner (2001) found significant nonlinear Granger causality from crude oil futures returns to S&P 500 index returns, and stock index returns also affect crude oil futures. Hammoudeh and Li (2005) suggested that there is a negative bidirectional dynamic relationship between crude oil price growth and the world capital market. On the contrary, some studies showed that that there is no significant relationship between oil shocks and stock markets (e.g., Al Janabi et al., 2010; Apergis and Miller, 2009; Henriques and Sadorsky, 2008). Recently, many papers have examined time-varying relationship between oil prices and stock returns (e.g., Chen, 2010; Chang and Yu, 2013; Sim and Zhou, 2015; Inchauspe et al., 2015; Kang et al., 2015), and the evidences of time-varying relation have been found.

The rest of the paper is organized as follows. In section 2, the ACP model with heteroskedastic regressors (i.e., ACP-H model) is introduced, and the model identification and estimation are examined. In the section 3, the model is used to examine time-varying effects of daily US dollar index and S&P 500 stock market on WTI crude oil price. The linear regression and ACP model are also applied. The in-sample and out-of-sample performances are compared. Section 4 concludes.

2. The autoregressive conditional parameter model with heteroskedastic regressor

Assuming that the vector of explanatory variables is i.i.d. with zero mean and positive definite covariance matrix, Lu and Wang (2016) proposed an ACP model.

$$\begin{aligned} & y_t = c_t + x_t'b_t + e_t,\ \ e_t|I_{t-1}^0 \sim F(0, \delta_t) \\ & \theta_t = \Omega + \textstyle\sum_{i=1}^{n} A_i\, z_{t-i} + \sum_{i=1}^{m} B_j\, \theta_{t-j}, \\ & \text{where}, \delta_t' = (\gamma_t', \varphi'), \theta_t' = (c_t, b_t', \gamma_t'), \text{and } z_t' = (y_t, V_x^{-1} y_t x_t', z_{v,t}') \end{aligned} \tag{1}$$

where, the error e_t is independent and follows the distribution F with zero-mean. $\delta_t = (\gamma_t', \varphi')$ is a vector of time-varying conditional parameters in the error distribution, where γ_t is the time-varying part and φ is the constant one. $x_t = (x_{1,t}, \dots, x_{k,t})'$ is the vector of explanatory variables with mean zero and positive definite covariance matrix V_x, and it's also independent from e_t. $\theta_t' = (c_t, b_t', \gamma_t')$ is a K*1 time-varying conditional parameter vector, which is conditioned on past information. K is the dimension of θ_t. c_t is the conditional intercept, and b_t is the conditional coefficient vector, so (c_t, b_t') is the vector of conditional parameters in the mean equation. $z_t = (y_t, V_x^{-1} y_t x_t', z_{v,t}')' = (z_{m,t}', z_{v,t}')'$ meets the following condition.

$$\begin{aligned} & E(z_{m,t}'|I_{t-1}) = E((y_t, V_x^{-1} y_t x_t')|I_{t-1}) = (c_t, b_t') \\ & E(z_{v,t}|I_{t-1}^0) = \gamma_t \end{aligned} \tag{2}$$

where, $I_{t-1} = (x_{t-1}, x_{t-2}, \dots; e_{t-1}, e_{t-2}, \dots)$ is the information set available at time t-1, and $I_{t-1}^0 = (I_{t-1}, x_t)$. Hence, z_t can be viewed as the sample observation of time-varying parameter vector θ_t. When γ_t is the widely-used conditional variance h_t, $z_{v,t} = e_t^2$. Finally, Ω is a $K \times 1$ parameter vector, and A_i and B_j are the $K \times K$ parameter matrices, which are usually assumed to be diagonal so that the curses of dimensionality can be avoided.

However, the presence of ARCH effect in explanatory variable violates the i.i.d. assumption and may affect the performance of ACP model substantially. A direct effect is that z_t in the ACP model (1) does not meet the condition (2). Because $c_t \in I_{t-1}$ and x_t is independent from I_{t-1}, the conditional expectation of $V_x^{-1} y_t x_t$ is

$$E(V_x^{-1} y_t x_t | I_{t-1}) = V_x^{-1} E(x_t(c_t + x_t'b_t + e_t)|I_{t-1}) = V_x^{-1} E(x_t x_t' b_t | I_{t-1}) = V_x^{-1} H_t b_t \tag{3}$$

$H_t = E(x_t x_t' | I_{t-1})$ is the conditional covariance matrix of x_t, which is not constant when the

ARCH effect exists in x_t. Thus, $E(V_x^{-1} y_t x_t | I_{t-1}) = b_t$ does not hold.

A new process z_t satisfying the condition (2) should be defined when explanatory variables present ARH effects. This paper supposes that γ_t is the conditional variance h_t. Assume that x_t is an independent vector with mean zero and time-varying conditional covariance matrix H_t, i.e., $E(x_t | I_{t-1}) = 0$ and $E(x_t x_t' | I_{t-1}) = H_t$. Then, z_t is defined as

$$z_t = \left(y_t, z_{b,t}', e_t^2\right)', \text{where } z_{b,t} = H_t^{-1} y_t x_t \text{ and } H_t = E\left(x_t x_t' \middle| I_{t-1}\right) \tag{4}$$

The conditional expectation of $z_{b,t}$ is

$$E\left(z_{b,t} \middle| I_{t-1}\right) = E\left(H_t^{-1} x_t \left(x_t' b_t + e_t\right) \middle| I_{t-1}\right) = E\left(H_t^{-1} x_t x_t' b_t \middle| I_{t-1}\right) = b_t \tag{5}$$

because $E(x_t e_t | I_{t-1}) = 0$ and $E\left(x_t x_t' \middle| I_{t-1}\right) = H_t$. Similar to Lu and Wang (2016),

$$E(y_t | I_{t-1}) = c_t, \; E\left(e_t^2 \middle| I_{t-1}^0\right) = h_t \tag{6}$$

Therefore, z_t defined by (4) satisfies the condition (2).

Then, the ACP model with heteroskedastic regressors is defined as

$$\begin{aligned} & y_t = c_t + x_t' b_t + e_t, \; e_t | I_{t-1}^0 \sim F(0, \delta_t) \\ & \theta_t = \Omega + \textstyle\sum_{i=1}^n A_i \, z_{t-i} + \sum_{i=1}^m B_j \, \theta_{t-j}, \\ & \text{where, } \delta_t' = \left(\gamma_t', \varphi'\right), \theta_t' = \left(c_t, b_t', \gamma_t'\right), \text{and } z_t' = \left(y_t, H_t^{-1} y_t x_t', z_{v,t}'\right) \end{aligned} \tag{7}$$

where, H_t is the conditional covariance matrix of x_t and can be estimated by a DCC GARCH model. When x_t is univariate, H_t is the conditional variance of x_t and can be estimated by a GARCH model.

Furthermore, if A_i and B_j are assumed to be block diagonal, and e_t is normal distributed, the ACP-H model can be rewritten as

$$\begin{aligned} & y_t = c_t + x_t' b_t + e_t, \;\; e_t | I_{t-1}^0 \sim N(0, h_t) \\ & c_t = \omega_0 + \textstyle\sum_{i=1}^m \beta_{0,i} \, c_{t-i} + \sum_{j=1}^n \alpha_{0,j} \, y_{t-j} \\ & b_t = \Omega^b + \textstyle\sum_{i=1}^m B_i^b \, b_{t-i} + \sum_{j=1}^n A_j^b \, z_{m,t-j}^b, \text{where } z_{m,t}^b = H_t^{-1} y_t x_t \\ & h_t = \alpha_0 + \textstyle\sum_{i=1}^p \beta_i h_{t-i} + \sum_{j=1}^q \alpha_j e_{t-j}^2 \end{aligned} \tag{8}$$

In model (8), the interactions between c_t, b_t and h_t are ignored. When the number of explanatory variables is very large, there may be the ‘curses of dimensionality' problem, so A_i^b and B_j^b can also be assumed to be diagonal in the empirical studies.

The conditional covariance matrix H_t is estimated before the ACP-H model is built. When x_t is univariate, H_t is its conditional variance. Then, a GARCH model can be applied for x_t, and $z_{m,t}^b$ in (8) is estimate by

$$z_{m,t}^b = \sigma_t^{-2} y_t x_t, \tag{9}$$

σ_t^2 is the conditional variance of x_t, which is estimated by a GARCH model. When x_t is multivariate, a multivariate GARCH model can be applied. This paper uses the DCC GARCH model of Engle (2002).

$$
\begin{aligned}
&x_t|I_{t-1} \sim N(0, D_t R_t D_t) \\
&D_t^2 = \mathrm{diag}\{\omega_i\} + \mathrm{diag}\{\kappa_i\} \circ x_{t-1}x_{t-1}' + \mathrm{diag}\{\lambda_i\} \circ D_{t-1}^2, \\
&\varepsilon_t = D_t^{-1} x_t \\
&Q_t = S \circ (\iota\iota' - A - B) + A \circ \varepsilon_{t-1}\varepsilon_{t-1}' + B \circ Q_{t-1} \\
&R_t = \mathrm{diag}\{Q_t\}^{-1} Q_t \, \mathrm{diag}\{Q_t\}
\end{aligned} \tag{10}
$$

where, I_t is the information available at time t, N is the normal distribution, and $H_t = D_t R_t D_t$ is the conditional covariance matrix of x_t. When the explanatory variables are mutually independent, the estimation of H_t is very simple. A GARCH model (e.g., Bollerslev, 1986) can be applied for each explanatory variable, and H_t can be estimated by

$$
H_t = \begin{pmatrix} \sigma_{1,t}^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{k,t}^2 \end{pmatrix} \tag{11}
$$

where, $\sigma_{l,t}^2$ is the conditional variance of $x_{l,t}$ and is estimated by a GARCH model.

The identification of ACP-H model is the same as that of Lu and Wang (2016) or ARMA process. The autocorrelation and partial autocorrelation functions for each element of z_t are calculated. Then, the characters of theoretical ACF and PACF for stationary ARMA process are used to identify the lag order (e.g., Wei, 1989; Lu and Wang, 2016). Furthermore, the ACP(1,1)-H model may be preferable in the empirical study if the sample ACFs and PACFs decay to zero at a large lag, which is similar to the popular GARCH(1,1) process.

Maximum likelihood estimation (MLE) is applied for the ACP-H model (8). Let L be the average of log-likelihood function and l_t be the log-likelihood function of t-th observation apart from the constant.

$$
L = \frac{1}{T}\sum_{t=1}^{T} l_t
$$

$$
l_t = -\frac{1}{2}\log(h_t) - \frac{e_t^2}{2h_t} \tag{12}
$$

The ML estimate can be obtained by maximizing L over the parameters.

Under certain conditions, the ML estimate exits and asymptotically follows a normal distribution (e.g., Greene, 2003; Crowder, 1976). When the error is not normal distributed, the quasi-maximum likelihood estimation (QMLE) can be applied (Bollerslev and Wooldridge, 1992). It’s very complex to identify these conditions, so this paper assumes that the ML estimator exits and is asymptotically normal distributed.

3. Empirical study

In this section, the ACP-H model is applied to study time-varying effects of US dollar index and S&P 500 stock index on WTI crude oil price. For the purpose of comparison, the traditional linear regression and the ACP model of Lu and Wang (2016) are also built. The in-sample and out-of-sample performances are examined.

The daily returns of WTI crude oil price (wti), US dollar index (dollar) and S&P 500 stock index (spx) are used in the paper. The return is the log-return $r_t = 100 * (\ln(P_t) - \ln(P_{t-1}))$, where P_t is the closing price. The whole sample is April 5, 1983 - September 30, 2015, and has 7933 observations. The data before December 30, 2012 is used for model estimation, and the remaining data (692 observations) is used to check the out-sample performance.

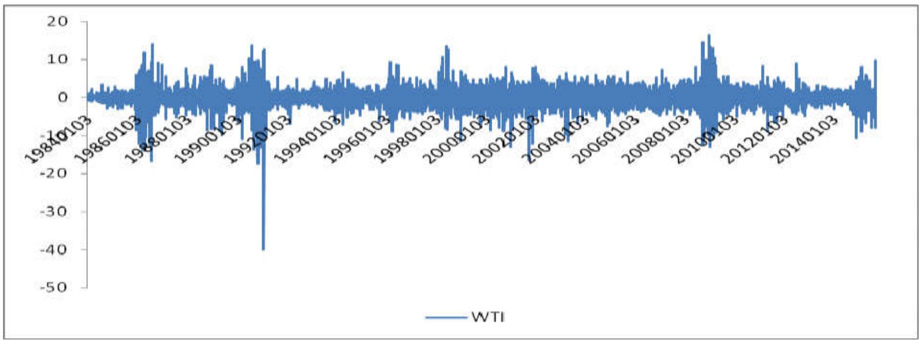


Fig. 1. The return of WTI crude oil price

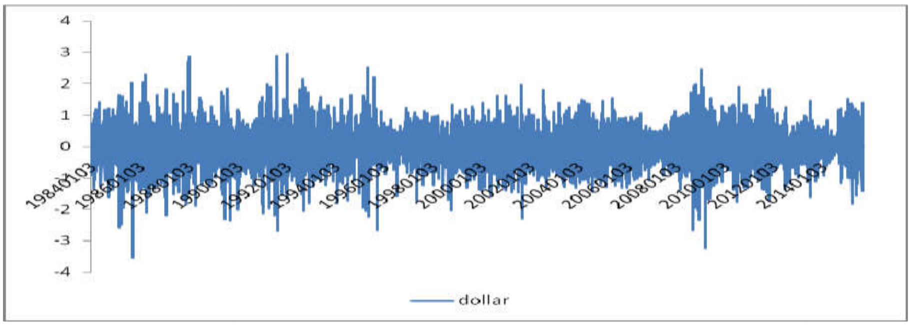


Fig. 2. The return of US dollar index

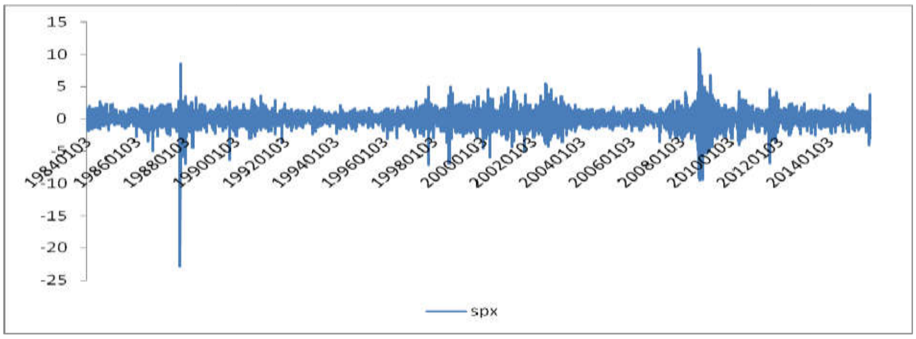


Fig. 3. The return of S&P 500 stock index

Fig.s 1-3 are the returns of WTI crude oil price, US dollar index, and S&P500 stock index. Each return exhibits the volatility clustering, which indicates the ARCH effect. For instance, the WTI return (Fig. 1) presents large changes when the 1990 Gulf Crisis and the 2008

Global Economic Crisis happened. Outliers can also be found[2]. Take the WTI return as an example, the sample mean and standard deviation are 0.01 and 2.40. Its value at Jan 17, 1991 is -40, which is smaller than 4 standard deviations, so it is an outlier. Similarly, we find the presences of outliers in US dollar and S&P 500 stock returns.

Because the outliers may affect the estimate and performance of a model, each return is cut off by four times the standard deviation apart from the mean. Then, each return is standardized by $r_t - \bar{r}$, because the explanatory variables in ACP model should be zero-mean. $\bar{r}$ is the sample mean of r_t.

Table 1 reports the Augmented Dickey-Fuller (ADF) unit root test and Engle's (1982) ARCH test. The ADF test statistics for wti, dollar and spx are -90.93, -89.48, and -91.62, which are all significant at 1% level. The ADF test rejects the unit root null hypothesis, so each process is stationary. The ARCH test shows that each process presents the ARCH effect significantly, so the ACP-H model may be more suitable. The presence of ARCH effect in wti return indicates that the error variance follows a GARCH model. Furthermore, the sample ACF, PACF and Ljung-Box Q statistics show that each return has non-significant or very limited autocorrelations[3], so the vector of explanatory variables is assumed to be independent in our empirical research[4]. In addition, the dependent variable wti has very weak autocorrelations[5], so the time-varying intercept in ACP and ACP-H models is assumed to be constant, i.e., $c_t = c$.

Table 1. The Augmented Dickey-Fuller (ADF) and ARCH tests

Tests	The ADF test	The ARCH test
wti	-90.93 [0.0001]	242.01 [0.0000]
dollar	-89.48 [0.0001]	49.82 [0.0000]
spx	-91.62 [0.0001]	403.92 [0.0000]

Notes: The value in [ ] is the P-value, and the lag of Engle ARCH test is 5.

The sample ACF and PACF of z_t are used for the model identification. Denote $z_t = H_t^{-1} y_t x_t = (zwd_t, zws_t)'$ in the ACP-H model, where $x_t = (dollar_t, spx_t)'$ and H_t is estimated by the DCC-GARCH model of Engle (2002)[6]. Let $z_t = (zwd0_t, zws0_t)'$ in the

[2] See Sincich (1986), Younger (1979) and Shiffler (1988).
[3] The sample ACF, PACF and Ljung-Box Q tests are classical, so the paper does not show the results, which can be obtained upon request.
[4] When there are strong autocorrelations, the ACP model may be affected substantially. Therefore, the ARMA model can be applied for explanatory variable to remove the autocorrelations before the ACP model is applied. The limited autocorrelations would not affect our empirical results, so they are ignored for simplicity in this paper.
[5] The absolute values of sample ACF and PACF for wti are smaller than 0.039, so the autocorrelation is very weak. Furthermore, the autocorrelation is not significant after the GARCH (1,1) model is applied, so the weak autocorrelation induced by the ARCH effect can be ignored.
[6] The estimation of Engle's (2002) DCC-GARCH model applies the "ccgarch" R package. The estimates are not

ACP model, where V_x^{-1} is estimated by the sample covariance matrix of x_t.

Fig. 4 shows the correlograms of zwd_t and zws_t, and Fig. 5 shows that of $zwd0_t$ and $zws0_t$. Significant autocorrelations are found, which indicates that the coefficients of dollar and spx may be time-varying or present ACP effects. The sample ACFs and PACFs decay to zero for large lags, so the order n in ACP(0,n) models may be large and there may be too many parameters. Take zwd_t for an example, the Q statistics shows that it has significant autocorrelations, its sample ACFs and PACFs are not zero even after the lag 10 (see the left part in Fig. 4), and the sample ACF and PACF at lag 13 are 0.022 and 0.017, which are still significant (see "AC" in the left part of Fig. 4). Therefore, we choose the order (0,13) for dollar in the ACP(0,n)-H model. Similarly, we choose the order (0,11) for spx in the ACP(0,n)-H model by the sample ACFs and PACFs (see the right part in Fig. 4). From Fig. 5, a larger lag n may be selected for each return, as the sample ACFs and PACFs usually decay to zero at a larger lag. For simplicity, we choose the same lag n in the ACP model as that in ACP-H model for each regressor[7]. Furthermore, it's shown that the sample ACF and PACF decay slowly. Therefore, the ACP(1,1) and ACP(1,1)-H models are applied, as they not only allow for the slow decay but also reduce the number of parameters. In summary, the selected orders for ACP and ACP-H models are shown in Table 2.

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.031	0.031	7.7915	0.005
		2	0.043	0.042	22.446	0.000
		3	0.013	0.011	23.811	0.000
		4	0.055	0.053	48.022	0.000
		5	0.005	0.001	48.234	0.000
		6	0.033	0.029	57.018	0.000
		7	0.012	0.009	58.110	0.000
		8	0.023	0.017	62.354	0.000
		9	0.035	0.032	71.879	0.000
		10	-0.019	-0.026	74.624	0.000
		11	0.004	0.001	74.756	0.000
		12	0.035	0.033	84.624	0.000
		13	0.022	0.017	88.592	0.000
		14	-0.001	-0.004	88.608	0.000
		15	-0.011	-0.016	89.590	0.000

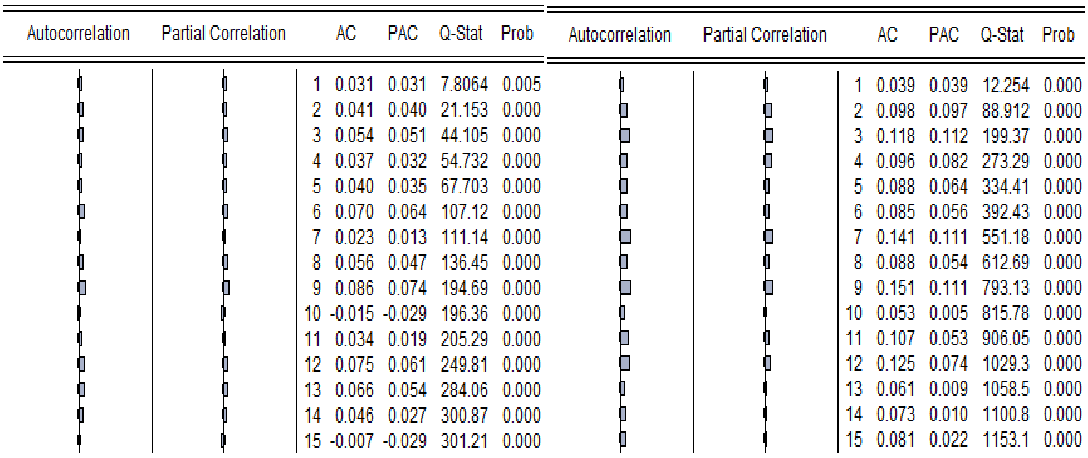
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.057	0.057	25.368	0.000
		2	0.068	0.065	62.392	0.000
		3	0.051	0.044	83.268	0.000
		4	0.035	0.026	93.213	0.000
		5	0.044	0.035	108.51	0.000
		6	0.037	0.027	119.27	0.000
		7	0.022	0.011	123.15	0.000
		8	0.056	0.046	147.68	0.000
		9	0.050	0.039	167.81	0.000
		10	0.038	0.023	179.17	0.000
		11	0.031	0.016	186.63	0.000
		12	0.013	-0.001	187.98	0.000
		13	0.021	0.009	191.44	0.000
		14	0.006	-0.005	191.73	0.000
		15	0.018	0.009	194.42	0.000

Fig. 4. Correlograms of zwd_t and zws_t

Notes: "AC" and "PAC" are the sample ACFs and PACFs, and "Q-stat" is the Ljung-Box Q statistics and "Prob" is the corresponding p-value.

shown in the paper, but they can be obtained upon request.

[7] A larger lag n can be selected from the sample ACFs and PACFs, but it only yields a limited improvements of the ACP(0,n) model. Therefore, we do not consider other orders for the ACP(0,n) model.



	AC	PAC	Q-Stat	Prob
1	0.031	0.031	7.8064	0.005
2	0.041	0.040	21.153	0.000
3	0.054	0.051	44.105	0.000
4	0.037	0.032	54.732	0.000
5	0.040	0.035	67.703	0.000
6	0.070	0.064	107.12	0.000
7	0.023	0.013	111.14	0.000
8	0.056	0.047	136.45	0.000
9	0.086	0.074	194.69	0.000
10	-0.015	-0.029	196.36	0.000
11	0.034	0.019	205.29	0.000
12	0.075	0.061	249.81	0.000
13	0.066	0.054	284.06	0.000
14	0.046	0.027	300.87	0.000
15	-0.007	-0.029	301.21	0.000

	AC	PAC	Q-Stat	Prob
1	0.039	0.039	12.254	0.000
2	0.098	0.097	88.912	0.000
3	0.118	0.112	199.37	0.000
4	0.096	0.082	273.29	0.000
5	0.088	0.064	334.41	0.000
6	0.085	0.056	392.43	0.000
7	0.141	0.111	551.18	0.000
8	0.088	0.054	612.69	0.000
9	0.151	0.111	793.13	0.000
10	0.053	0.005	815.78	0.000
11	0.107	0.053	906.05	0.000
12	0.125	0.074	1029.3	0.000
13	0.061	0.009	1058.5	0.000
14	0.073	0.010	1100.8	0.000
15	0.081	0.022	1153.1	0.000

Fig. 5. Correlograms of $zwd0_t$ and $zws0_t$

Table 2. Orders of the ACP-H and ACP models for dollar and spx

Explanatory variables	dollar	spx
Lag orders	(0,13) and (1,1)	(0,11) and (1,1)

Table 3 reports the estimates of linear regression, ACP(1,1) and ACP(1,1)-H models[8]. The first column is the linear regression. The coefficient of dollar is -0.240192, which is significant at 5% level, so dollar has a negative effect on wti. The coefficient of spx is 0.124348 and is also significant, which implies that spx has a positive effect on wti. Therefore, the US dollar index has a negative effect on WTI crude oil price, but the S&P 500 stock index has a positive effect on WTI crude oil price. Besides, the ARCH effect is found from the estimates of e_{t-j}^2 and h_{t-1}.

The estimates of ACP(1,1) and ACP(1,1)-H models are shown in the 2nd and 3rd columns in Table 3. $b_{1,t}$ and $b_{2,t}$ are the time-varying parameter of dollar and spx, respectively. From the estimates of ACP(1,1)-H model, the estimated parameters of $b_{1,t-1} * \text{dollar}$ and $z_{1,t-1} * \text{dollar}$ are 0.9946 and 0.0029. They are all significant at 5% level, which indicates the presentence of ACP effect, and the sum is close to 1, which means that time-varying parameter $b_{1,t}$ is strongly persistent. From the estimates of $b_{2,t-1} * \text{spx}$ and $z_{2,t-1} * \text{spx}$, a strong ACP effect is found and the persistent time-varying parameter $b_{2,t}$ is shown, too. Furthermore, the mean values of $b_{1,t}$ and $b_{2,t}$ are -0.2237 and 0.00543 in turn. Therefore, the US dollar has a negative effect on wti, and spx has a positive effect, which is similar to that from the linear regression. Similarly, we draw similar conclusions from the estimates of ACP(1,1) model (the 2nd column in Table 3). Finally, the R^2 values of three models indicate that ACP(1,1)-H model has the best performance, and ACP(1,1) model also outperforms the linear regression.

Table 3. Estimates of linear regression, ACP(1,1) and ACP(1,1)-H models

[8] The program for ACP(1,1) and ACP(1,1)-H models is written in the GAUSS language and shown in the appendix.

Variables	Linear regression	ACP(1,1)	ACP(1,1)-H
c	0.0113 [0.5956]	0.0076 [0.3918]	0.0101 [0.5523]
dollar	-0.2402 [-8.0799]	-0.0002 [-0.6667]	0.0001 [0.5000]
$z_{1,t-1} * \text{dollar}$	--	0.0020 [2.8571]	0.0077 [4.8125]
$b_{1,t-1} * \text{dollar}$	--	0.9941 [523.21]	0.9893 [380.50]
spx	0.1243 [6.8965]	0.0000 [--]	-0.0001 [-0.5000]
$z_{2,t-1} * \text{spx}$	--	0.0013 [6.5000]	0.0033 [3.6667]
$b_{2,t-1} * \text{spx}$	--	0.9978 [1995.60]	0.9955 [765.76]
cons	0.0159 [6.3317]	0.0198 [4.1250]	0.0194 [4.1276]
e_{t-j}^2	0.0572 [19.771]	0.0668 [13.360]	0.0655 [13.100]
h_{t-1}	0.9423 [345.32]	0.9320 [198.40]	0.9337 [198.66]
R^2	0.0120	0.0492	0.0583

Notes: “c” is the intercept in mean equation, "cons" is that in variance equation, and the value in [ ] is the t-value. “--” denotes N.A., and R^2 is the R-squared statistics.

For the ACP(0,n) and ACP(0,n)-H models, they include too many parameters and are usually inferior to the ACP(1,1)-H model in our empirical research, so their estimates are not shown in the paper. But their performances are examined below. Similarly, the linear regression, ACP and ACP-H models with one regressor are built to check the performances, but the estimates are not shown, too.

Table 4 shows the root-mean-square error (RMSE), mean absolute deviation (MAD) and R^2 of the models. From the statistics for in-sample fit, the ACP(1,1)-H model has the best performance, because its R^2 is the largest and its RMSE and MAD values are the smallest. It’s also shown that the ACP(0,n)-H model outperforms the corresponding ACP(0,n) model. Furthermore, the ACP-H and ACP models are obviously better than the linear regression. Take the R^2 statistics for an instance. The R^2 of multiple linear regression for the in-sample period is only 0.0120, but these of ACP(0,n), ACP(0,n)-H, ACP(1,1) and ACP(1,1)-H models with two explanatory variables are 0.0433, 0.0527, 0.0492 and 0.0583, which are higher than that of multiple linear regression. Furthermore, the ACP-H model has a better performance than ACP model. From the out-of-sample test statistics, we can draw the similar conclusions. In summary, the ACP(1,1)-H model is almost the best, and the ACP(0,n)-H model usually has an edge over the corresponding ACP(0,n) model.

Table 4. The performances of linear regression, ACP model and ACP-H model

Regressors	Models	In-sample			Out-of-sample		
		RMSE	MAD	R^2	RMSE	MAD	R^2
dollar	Linear regression	2.3027	1.6272	0.0077	1.9571	1.3541	0.0138
	ACP(0,n)	2.2818	1.6139	0.0257	1.9354	1.3413	0.0355
	ACP(0,n)-H	2.2802	1.6144	0.0271	**1.9319**	**1.3407**	**0.0391**
	ACP(1,1)	2.2772	1.6044	0.0279	1.9501	1.3533	0.0209
	ACP(1,1)-H	**2.2739**	**1.6021**	**0.0308**	1.9516	1.3610	0.0193
spx	Linear regression	2.3070	1.6300	0.0041	1.9500	1.3459	0.0209
	ACP(0,n)	2.2800	1.6087	0.0272	1.9340	1.3384	0.0370
	ACP(0,n)-H	2.2688	1.6041	0.0367	1.9395	1.3327	0.0315
	ACP(1,1)	2.2589	1.5920	0.0435	1.9189	1.3191	0.0519
	ACP(1,1)-H	**2.2579**	**1.5951**	**0.0444**	**1.9059**	**1.3107**	**0.0647**
both	Linear regression	2.2978	1.6210	0.0120	1.9328	1.3332	0.0382
	ACP(0,n)	2.2610	1.5974	0.0433	1.9066	1.3206	0.0641
	ACP(0,n)-H	2.2499	1.5933	0.0527	1.9001	1.3065	0.0704
	ACP(1,1)	2.2521	1.5856	0.0492	1.9043	1.3051	0.0664
	ACP(1,1)-H	**2.2413**	**1.5818**	**0.0583**	**1.8736**	**1.2842**	**0.0974**

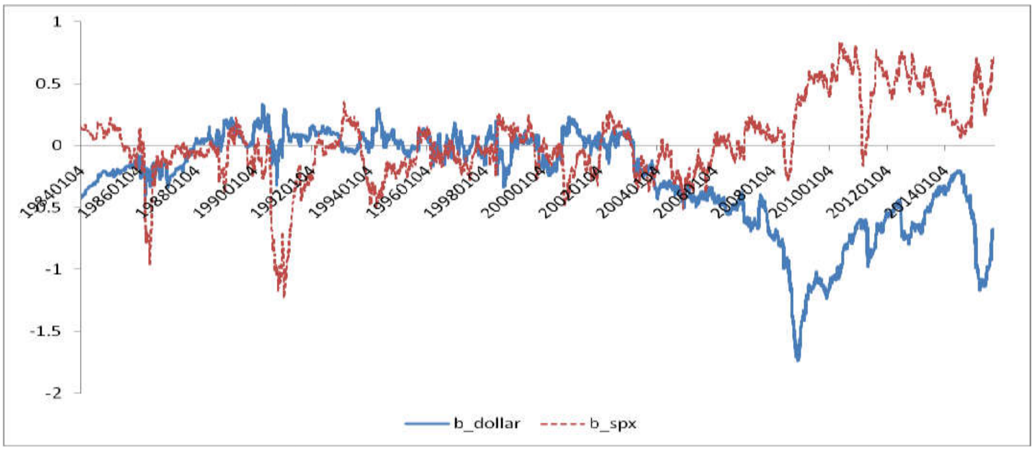


Fig. 6. Time-varying effects of US dollar and S&P 500 stock on crude oil price

Note: "b_dollar" is the time-varying coefficient of US dollar, "b_spx" is that of S&P 500 stock, and they are estimated by the ACP(1,1)-H model.

Finally, the ACP(1,1)-H model with two explanatory variables (see the ACP(1,1)-H model in Table 3) is used to examine time-varying causal effects of US dollar index and S&P 500 stock index on WTI crude oil price. The time-varying coefficients of dollar and spx are shown in Fig. 6. The time-varying coefficient of dollar is usually smaller than 0 (see "b_dollar" in Fig. 6), which implies that US dollar index usually depresses WTI crude oil price. But sometimes it is larger than 0, which indicates a positive effect of US dollar index on WTI crude oil price. For instances, in the subperiods of November 22, 1989 - July 31, 1990; August 20, 1991 - January 4, 1993; January 3, 1994 - July 11, 1994, and September 22, 2000 - June 27, 2001, it is usually positive. In addition, it's negative after March 2003 and becomes much smaller after December 2007. Thus, the US dollar index has a stronger negative impact on WTI crude oil after December 2007 averagely, which means that US monetary policy played a more important role in global crude oil pricing.

Meanwhile, the time-varying coefficient of spx is usually larger than 0 (see "b_spx" in Fig. 6), which means that the S&P 500 stock index has a positive effect on WTI crude oil price on the average. But it has a negative influence on WTI crude oil price in some subperiods (e.g., August 3, 1990 - June 27, 1991; September 21, 1993 - August 31, 1995; February 10, 2003 - February 27, 2004). Furthermore, the time-varying coefficient of spx is usually larger than 0.25 and rises to 0.918 at August 29, 2011, so the S&P 500 stock index has a stronger effect on WTI crude oil market after early 2009.

In a word, our empirical study shows the evidences of time-varying effects of US dollar and stock market on WTI crude oil price. The findings may be helpful for understanding the evolving crude oil pricing and improving the prediction accuracy.

4. Conclusion

To do with heteroskedasticity or ARCH effect in financial time series, this paper extends the autoregressive conditional parameter (ACP) model of (Lu and Wang, 2015) to the ACP model with heteroskedastic regressors. The conditional covariance matrix of explanatory variables is firstly estimated by a dynamic conditional correlation (DCC) - GARCH model. Then, the ACP-H model can be built and maximum likelihood estimation is used to solve the parameters.

The ACP-H, ACP and linear regression models are used to examine time-varying effects of US dollar index and S&P 500 stock index on WTI crude oil price. The data of April 5, 1983 - December 30, 2012 is used to estimate the models, and that of January 3, 2013 - September 30, 2015 is used to check the out-of-sample performances. From the root-mean-square error (RMSE), mean absolute deviation (MAD) and R^2, the ACP-H model improves the in-sample and out-of-sample performances substantially. For examples, the in-sample R^2 of ACP(1,1)-H model with two explanatory variables is 0.0583, which is the largest, and the out-of-sample one is 0.0974 and is also the largest.

The empirical results show that the coefficients of US dollar and S&P 500 stock returns in the ACP-H model change with time significantly, which indicates that their influences on WTI crude oil price are time-varying. Both positive and negative effects of US dollar and stock market on WTI crude oil price are found, which accommodates the complex findings of previous empirical studies. Meanwhile, their influences on crude oil price are enhanced substantially after 2008. Specially, the US dollar index has a stronger negative impact on WTI crude oil after December 2007, which implies that US monetary policy played a more important role in crude oil pricing thereafter.

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Appendix (Gauss code for ACP(1,1)-H model)

```
/*
**   ACP(1,1) model-garch(1,1) allows explanatory variables to be heteroskedastic.
**   x(t) is a vector of two elements, which follows the dynamic conditional correlation GARCH model of Engle (2002).
**   _cml_Algorithm   -    scalar, indicator for optimization method:
**                         = 1,     BFGS (Broyden, Fletcher, Goldfarb, Shanno)
**                         = 2,     DFP (Davidon, Fletcher, Powell)
**                         = 3,     NEWTON (Newton-Raphson)
**                         = 4,     BHHH
*/

new;

load data[7934,6]=e:\program\data\wtispxdollar_acp_dccgarch.txt;
@ date, wti,spx,dollar,Zspx_dcc Zdollar_dcc; @
@ In the ACP-H model, the DCC-GARCH model is used to estimate dynamic conditional covariance of x(t) @

tim=data[2:rows(data),1];
r0=data[2:rows(data),2:6];
N0=rows(r0); @ length of data @

/*
a1=acf(r0[.,1],20,0);
a2=acf(r0[.,2],20,0);
a3=acf(r0[.,3],20,0);
print a1~a2~a3;
pa1=pacf(r0[.,1],20,0);
print pa1;
*/
@ ACP(1,1) model. The error follows a GARCH(1,1) process @
p=1;
q=1;
garchp=1;garchq=1;

s=1; @[s:e] is the in-sample period@
e=7241;
r=r0[s:e,.];
N=rows(r);

/* The process yt and xt are zero-mean */
r1=r[.,1]; @ r1=r1-meanc(r1); @
r2=r[.,2]; @ r2=r2-meanc(r2); @
```

```
r3=r[.,3]; @ r3=r3-meanc(r3); @
z1=r[.,4];
z2=r[.,5];

x=r1~r2~r3;    @ x is the input data used in MLE @

library cml,pgraph;
#include cml.ext;
#include pgraph.ext;
cmlset;
graphset;

 /* The initial values and constraints of parameters.    */

   numx0=2; @ number of explanatory variables @
   @ the number of paramters in the mean equation: [ 1+numx*(p+q+1)+ (1+garchp+garchq) ]    @
   @ 1+numx0*(p+q+1) is the number of parameter in mean equation @
   @ (1+garchp+garchq) is the number of parameter in garch(p,q) equation @
   nm0=numx0*(p+q+1); @ number of parameter in the ACP equation @

   @ get the initial value of params: @

        bx1=olsqr(x[.,1],x[.,2]);
        bx2=olsqr(x[.,1],x[.,3]);

            @ b0=zeros(1+numx0*(p+q+1),1)|0.05*stdc(x[.,1])^2|0.05*ones(garchq,1)|0.9; b0[1]=bxar; @

             b0=0|(0.05*bx1)|0.01|0.95|(0.05*bx2)|0.01|0.95|0.05*stdc(x[.,1])^2|0.05|0.9;
            @ b0=0|0.05*bx1|0.05|0.9|0.05*bx2|0.05|0.9|0.05*stdc(x[.,1])^2|0.05|0.9; @

            _ww_ = { -1e256 1e256 };
            _cml_Bounds=ones(1+nm0+garchp+garchq+1,2).*_ww_;
            _cml_Bounds[3:p+q+2,1]=-0.99999*ones(p+q,1);
            _cml_Bounds[3:p+q+2,2]=0.99999*ones(p+q,1);

            _cml_Bounds[p+q+4:nm0+1,1]=-0.99999*ones(p+q,1);
            _cml_Bounds[p+q+4:nm0+1,2]=0.99999*ones(p+q,1);

            @    _cml_Bounds[2,1]=(1e-6)*ones(1,1);    _cml_Bounds[2,2]=0.99999*ones(1,1);@        @constraints    of    ar(1)
parameter@

                @nonegative parameter in variance equation @
            _cml_Bounds[2+nm0:1+nm0+garchp+garchq+1,1]=(1e-6)*ones(garchq+garchp+1,1);
            _cml_Bounds[3+nm0:1+nm0+garchp+garchq+1,2]=0.99999*ones(garchq+garchp,1);
```

```
        _cml_Algorithm=4;
        _cm_DirTol=1e-9;
        _cml_MaxIters=3000;
        format /rd 13,8;

        {b,f,g,h,retcode}=cmlPrt(cml(x,0,&acpll,b0));

print b h;

/*
calculate and output the dummy variable of out-of-sample, error, acp
*/

   x0=r0[.,1]~r0[.,2]~r0[.,3];
   z10=r0[.,4];
   z20=r0[.,5];
   N0=rows(x0);
   u0=zeros(N0,1);
   h0=zeros(N0,1);
   acp0=zeros(N0,2);
   duminsamp=zeros(N,1)|ones(N0-N,1);

   acp0[1,1]=olsqr(x0[.,1],x0[.,2]);
   acp0[1,2]=olsqr(x0[.,1],x0[.,3]);
   u0[1]=x0[1,1]-b[1]-acp0[1,1]*x0[1,2]-acp0[1,2]*x0[1,3];

 for i (2,N0,1);
            acp0[i,1]=b[2]+b[3]*z10[i-1]+b[4]*acp0[i-1,1];
            acp0[i,2]=b[5]+b[6]*z20[i-1]+b[7]*acp0[i-1,2];
            u0[i]=x0[i,1]-b[1]-acp0[i,1]*x0[i,2]-acp0[i,2]*x0[i,3];
        endfor;

output file=e:\program\data\output_acp11_2regressor.txt reset;
print    duminsamp~u0~acp0;
end output;

/*
   in the log-likelihood functuon acpll, p=q=garchp=garchq=1.
*/

proc acpll(b,data);    @ ACP(1,1)_garch_ normal distribution @
   local T,acp,numx,u,utemp,u2temp,ugarch,i,h,htemp,he,k,mx,Tg,ll;
   T=rows(data);
```

```
numx=cols(data)-1; @ the number of independent vars @
u=zeros(T,1);
h=zeros(T,1);
acp=zeros(T,2);

acp[1,1]=olsqr(data[.,1],data[.,2]);
acp[1,2]=olsqr(data[.,1],data[.,3]);
u[1]=data[1,1]-b[1]-acp[1,1]*data[1,2]-acp[1,2]*data[1,3];

for i (2,T,1);
        acp[i,1]=b[2]+b[3]*z1[i-1]+b[4]*acp[i-1,1];
        acp[i,2]=b[5]+b[6]*z2[i-1]+b[7]*acp[i-1,2];
        u[i]=data[i,1]-b[1]-acp[i,1]*data[i,2]-acp[i,2]*data[i,3];
    endfor;

/*
 **    h: Conditional variance vector:
 **    u : the residual
 **    calculate the conditional variance vector:
 **    alpha- garch(garchp,garchq) : b[p+q+numx+1: p+q+numx+1+garchp+garchq]
 */
    mx=maxc(garchp|garchq);
    ugarch= u[1:rows(u)]; @ the residual uesd in Garch model:delete the first p value @
    Tg=T;
    k=numx*(p+q+1)+2;    @ b[k:rows(b)] is the parameter vector in variance equation @

 if garchp==0 and garchq==0;      @ constant varince @
      he=b[k]*ones(Tg,1);

 elseif    garchp==0 and garchq>=1;      @ arch( garchq ) model @
      u2temp=(stdc(ugarch)^2)*ones(mx,1)|( ugarch^2 );
      htemp=(stdc(ugarch)^2)*ones(mx+Tg,1);
      for i (1,Tg,1);
         htemp[i+mx]=b[k]+ (u2temp[i+mx-1:i-garchq])'*b[k+1:k+garchq];
      endfor;
       he=htemp[mx+1:(mx+Tg)];

 elseif    garchp>=1 and garchq==0;       /* garch model */
      htemp=(stdc(ugarch)^2)*ones(mx+Tg,1);
      for i (1,Tg,1);
         htemp[i+mx]=b[k]+ htemp[(i+mx-1):(i+mx-garchp)]'*b[k+1:k+garchp];
      endfor;
      he=htemp[mx+1:mx+Tg];
```

```
    elseif    garchp>=1 and garchq>=1;        /* garch model */
               htemp=(stdc(ugarch)^2)*ones(mx+Tg,1);
                    @ htemp=(b[4]/(1-b[5]-b[6]))*ones(mx+Tg,1);@
               u2temp=( (stdc(ugarch)^2 )*ones(mx,1) )|ugarch^2;
             for i (1,Tg,1);
                 htemp[i+mx]=b[k]+(u2temp[i+mx-1:i+mx-garchq])'*b[k+1:k+garchq]+
                                    htemp[(i+mx-1):(i+mx-garchp)]'*b[k+garchq+1:k+garchq+garchp];
             endfor;
             he=htemp[mx+1:mx+Tg];

    endif;

   ll=-0.5*(ln(2*pi)+ln(he)+(ugarch^2)./he);
   retp(ll);

endp;
```